LCR Series Circuits

Introduction to LCR Series Circuits

What you'll learn in Module 9.

Module 9 Introduction

Introduction to LCR Series Circuits.

Section 9.1 LCR Series Circuits.

Recognise LCR Series circuits and describe their action using phasor diagrams and appropriate equations:

- Below Resonance
- Above Resonance
- At Resonance

Section 9.2 Series resonance.

- Describe LCR Series Circuits at resonance.
- Describe the conditions for series resonance.
- Carry out calculations on LCR series circuits, involving reactance, impedance, voltages and current.

Section 9.3 Voltage Magnification.

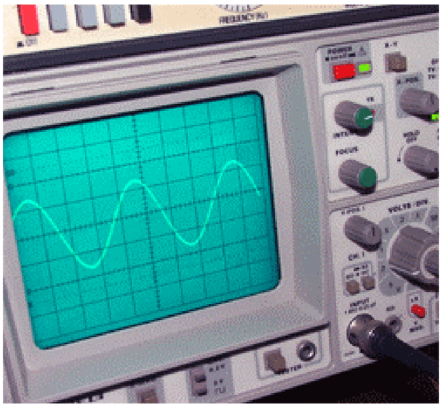
- Describe voltage magnification in LCR Series Circuits
- Calculate Voltage Magnification using appropriate formulae.

Section 9.4 LCR Series Quiz.

- LCR Series Circuits Quiz.

Amazing LCR Circuits.

This module introduces some of the most useful and most amazing circuits in electronics. They can be as simple as two or three components connected in series, but in their operation they can perform many complex tasks and are used perhaps, in more circuit applications than any other circuit arrangement.



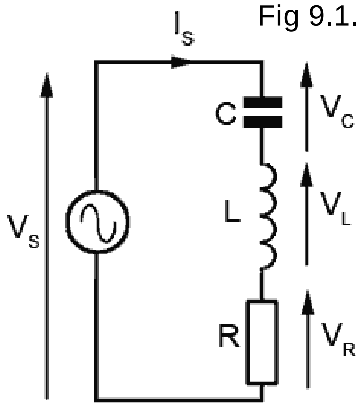
Connecting an inductor, a capacitor and perhaps a resistor, either in series or in parallel, makes some surprising things happen. Previous modules in this series have examined capacitors and inductors in isolation, and combined with resistors. These have created useful circuits such as filters, differentiators and integrators. Now module 9 looks at what happens when inductors and capacitors are combined in a single circuit network.

Capacitors and inductors act in different (and often opposite) ways in AC circuits. This module is about combining the properties of reactance and impedance of capacitors and inductors with varying frequency to produce amazing effects.

A circuit containing L,C and R at a certain frequency can make L and C (or at least their electrical effects) completely disappear! The LCR circuit can appear to be just a capacitor, just an inductor, or solely a resistor! Not only that, the series LCR circuit can magnify voltage, so the voltages across individual components within the circuit, can actually be much larger than the external voltage supplying the circuit. LCR circuits can also dramatically change their impedance to offer more or less opposition to current at different frequencies. All these effects can be used separately or together to make the wide range of electronic devices that use AC.

Module 9.1 LCR Series Circuits.

Fig 9.1.1



The circuit in Fig 9.1.1 contains all the elements so far considered separately in modules 1 to 8, namely inductance, capacitance and resistance, as well as their properties such as Reactance, Phase, Impedance etc.

This module considers the effects of L C and R connected together in series and supplied with an alternating voltage. In such an arrangement, the same circuit supply current (I_S) flows through all the components of the circuit, and V_R V_L and V_C indicate the voltages across the resistor, the inductor and the capacitor respectively.

Module 6.1 described the effect of internal resistance on the voltage measured across an inductor. In LCR circuits both internal (inductor) resistance, and external resistance are present in the complete circuit. Therefore, it will be easier to begin with, to consider that the voltage V_R is the voltage across the TOTAL circuit resistance, which comprises the internal resistance of L, added to any separate fixed resistor. Where V_S is mentioned, this is the applied supply voltage.

The phase relationship between the supply voltage V_S and the circuit current I_S depends on the frequency of the supply voltage, and on the relative values of inductance and capacitance, and whether the inductive reactance (X_L) is greater or less than the capacitive reactance (X_C). There are various conditions possible, which can be illustrated using phasor diagrams.

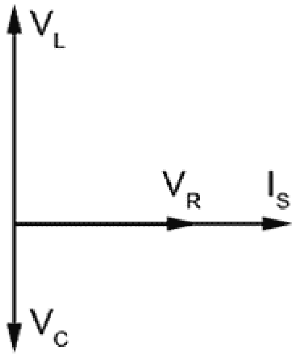


Fig 9.1.2 Phasors for V_L and V_C are in anti phase.

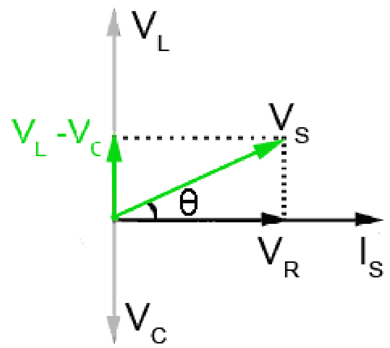


Fig 9.1.3 V_L is greater than V_C so the circuit behaves like an inductor

Fig 9.1.2 shows the circuit conditions when the inductive reactance (X_L) is greater than the capacitive reactance (X_C). In this case, since both L and C carry the same current, and X_L is greater than X_C, it follows that V_L must be greater than V_C.

$$\mathbf{V_L = I_S X_L \quad \text{and} \quad V_C = I_S X_C}$$

Remember that V_C and V_L are in anti-phase to each other due to their 90° leading and lagging relationship with the circuit current (I_S). As V_L and V_C directly oppose each other, a resulting voltage is created, which will be the difference between V_C and V_L. This is called the REACTIVE VOLTAGE and its value can be calculated by simply subtracting V_C from V_L. This is shown in Fig 9.1.3 by the phasor ($V_L - V_C$).

The length of the phasor ($V_L - V_C$) can be arrived at graphically by removing a portion from the tip of the phasor (V_L), equivalent to the length of phasor (V_C).

V_S is therefore the phasor sum of the reactive voltage ($V_L - V_C$) and V_R. The phase angle θ shows that the circuit current I_S lags on the supply voltage V_S by between 90° and 0°, depending on the relative sizes of ($V_L - V_C$) and V_R. Because I_S lags V_S, this must mean that the circuit is mainly inductive, but the value of inductance has been reduced by the presence of C. Also the phase difference between I_S and V_S is no longer 90° as it would be if the circuit consisted of only pure inductance and resistance.

Because the phasors for ($V_L - V_C$), V_R and V_S in Fig 9.1.3 form a right angle triangle, a number of properties and values in the circuit can be calculated using either Pythagoras´ Theorem or some basic trigonometry, as illustrated in "Using Phasor Diagrams" in Module 5.4.

For example:

$$V_S^2 = (V_L - V_C)^2 + V_R^2 \quad \text{therefore} \quad V_S = \sqrt{(V_L - V_C)^2 + V_R^2}$$

The total circuit impedance (Z) can be found in a similar way: The phase angle between ($V_L - V_C$) and

$$Z = \sqrt{(X_L - X_C)^2 + R^2}$$

V_R can be found using trigonometry as illustrated in "Using Phasor Diagrams" in Module 5.4.

tan θ = opposite ÷ adjacent, therefore **tan θ = (VL − VC) ÷ VR**

so to find the angle θ

$$\theta = \tan^{-1} \frac{(V_L - V_C)}{V_R}$$

Also, Ohms Law states that R (or X) = V / I

Therefore if ($V_L - V_C$) and V_R are each divided by the current (I_S) this allows the phase angle θ to be found using the resistances and reactances, without first working out the individual voltages.

$$\theta = \tan^{-1} \frac{(X_L - X_C)}{R}$$

This can be useful when component values need to be chosen for a series circuit, to give a required angle of phase shift.

When VC is larger than VL the circuit is capacitive.

Fig 9.1.4 illustrates the phasor diagram for a LCR series circuit in which X_C is greater than X_L showing that when V_C exceeds V_L the situation illustrated in Fig 9.1.3 is reversed.

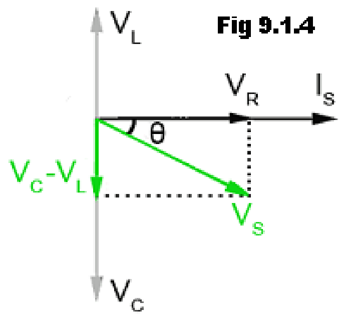


Fig 9.1.4

The resultant reactive voltage is now given by ($V_C - V_L$) and V_S is the phasor sum of ($V_C - V_L$) and V_R.

The phase angle θ now shows that the circuit current (I_S) leads supply voltage (V_S) by between 0° and 90°. The overall circuit is now capacitive, but less so than if L was not present.

In using the above formulae, remember that the reactive value (the difference between V_L and V_C or X_L and X_C) is given by subtracting the smaller value from the larger value. For example, when V_C is larger than V_L:

$$V_S = \sqrt{(V_C - V_L)^2 + V_R^{\ 2}}$$

Looking at the phasor diagrams for a LCR series circuit it can be seen that the supply voltage (V_S) can either lead or lag the supply current(I_S) depending largely on the relative values of the component reactances, X_L and X_C.

When VL and VC are equal the circuit is purely resistive.

As shown in Module 6.1 and 6.2, the reactance of L and C depends on frequency, so if the frequency of the supply voltage V_S is varied over a suitable range, the series LCR circuit can be made to act as either an inductor, or as a capacitor, but that's not all.

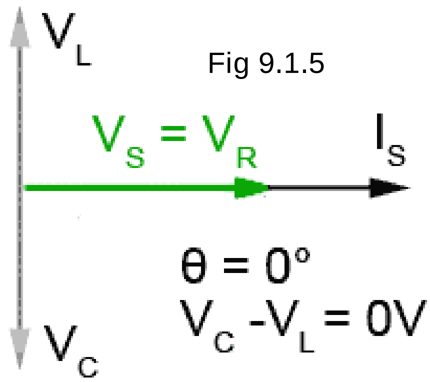


Fig 9.1.5

Fig 9.1.5 shows the situation, which must occur at some particular frequency, when X_C and X_L (and therefore V_C and V_L) are equal.

The opposing and equal voltages V_C and V_L now completely cancel each other out. **The supply voltage and the circuit current must now be in phase, so the circuit is apparently entirely resistive!** L and C have completely "disappeared".

This special case is called SERIES RESONANCE and is explained further in Module 9.2.

Module 9.2 Series Resonance

Series Resonance happens when reactances are equal.

Inductive reactance (X_L) in terms of frequency and inductance is given by:

$$X_L = 2\pi f L$$

and capacitive reactance (X_C) is given by:

$$X_C = \frac{1}{2\pi f C}$$

Inductive reactance is **directly proportional** to frequency, and its graph, plotted against frequency (f) is a straight line.

Capacitive reactance is **inversely proportional** to frequency, and its graph, plotted against f is a curve. These two quantities are shown, together with R, plotted against f in Fig 9.2.1 It can be seen from this diagram that where X_C and X_L intersect, they are equal and so a graph of ($X_L - X_C$) must be zero at this point on the frequency axis.

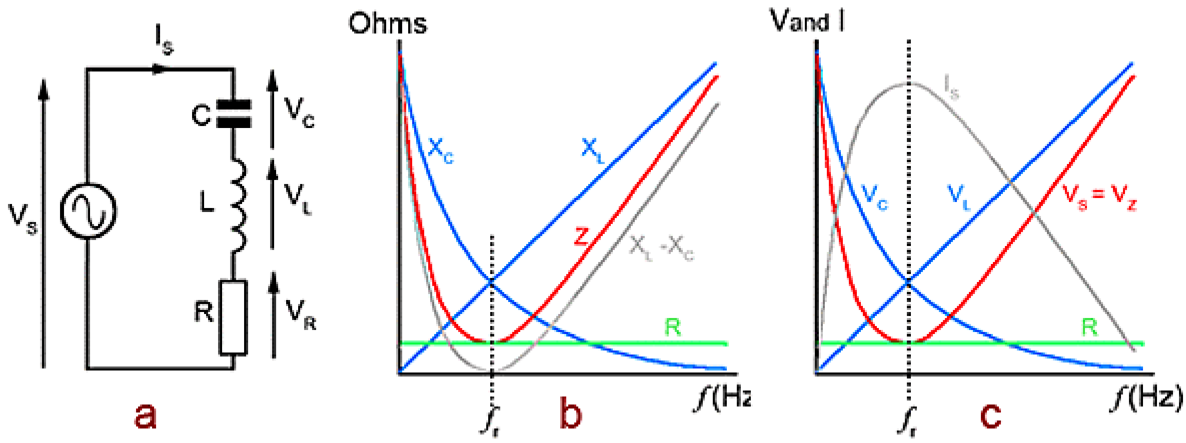


Fig 9.2.1 The Properties of a Series LCR Circuit at Resonance.

Fig 9.2.1a shows a series LCR circuit and Fig 9.2.1b shows what happens to the reactances (X_C and X_L), resistance (R) and impedance (Z) as the supply (V_S) is varied in frequency from 0Hz upwards. At first the circuit behaves as a capacitor, the total impedance of the circuit (Z) falls in a very similar curve to $X_L - X_C$.

Fig 9.2.1c illustrates the relationships between the individual component voltages, the circuit impedance(Z) and the supply current (I_S) (which is common to all the series components).

At a particular frequency f_r it can be seen that $X_L - X_C$ has fallen to zero and only the circuit resistance R is left across the supply. The current flowing through the circuit at this point will therefore be at a maximum. Now V_C and V_L are equal in value and opposite in phase, so will completely cancel each other out. Reactance is effectively zero and the circuit is completely resistive, with Z equal to R. The circuit current (I_S) will be at its maximum and will be in phase with the supply voltage (V_S) which is at its minimum.

As the frequency increases above this resonant frequency (f_r) the impedance rises, and as X_L is now the larger of the two reactances, the impedance curve begins to follow an increasing value more like the linear graph of X_L.

At frequencies below resonance the circuit behaves like a capacitor, at resonance as a resistor, and above f_r the circuit behaves more and more like an inductor, and the graph of $X_L - X_C$ soon becomes an almost straight line.

This behaviour of a LCR Series Circuit allows for the statement of a number of useful facts about a series circuit that relate to its resonant frequency f_r.

6 Things you need to know about LCR Series Circuits.

1. **AT RESONANCE (f_r)** V_C is equal to, but in anti-phase to V_L

2.; **AT RESONANCE (f_r)** Impedance (Z) is at minimum and equal to the RESISTANCE (R)

3. **AT RESONANCE (f_r)** Circuit current (I_S) is at a maximum.

4. **AT RESONANCE (f_r)** The circuit is entirely resistive.

5. **BELOW RESONANCE (f_r)** The circuit is capacitive.

6. **ABOVE RESONANCE (f_r)** The circuit is inductive.

Two Formulae for Series Resonance.

The fact that resonance occurs when $X_L = X_C$ allows a formula to be constructed that allows calculation of the resonant frequency (f_r) of a circuit from just the values of L and C. The most commonly used formula for the series LCR circuit resonant frequency is:

$$f_r = \frac{1}{(2\pi\sqrt{LC})}$$

Notice that this formula does not have any reference to resistance (R). Although any circuit containing L must contain at least some resistance, the presence of a small amount of resistance in the circuit at high frequencies does not greatly affect the **frequency** at which the circuit resonates. Resonant circuits designed for high frequencies are however, affected by stray magnetic fields, inductance and capacitance in their nearby environment. These enviromental issues have a greater effect on the resonant frequency than the small amount of internal resistance present. therefore most high frequency LC resonant circuits will have both screening (using some form of metal container) to isolate them from external effects as much as possible, and may also be made adjustable over a small range of frequency, so they can be accurately adjusted after assembly in the circuit.

Where does the formula for f_r come from?

The formula for finding the resonant frequency can be built from the two basic formulae that relate inductive and capacitive reactance to frequency.

$$X_L = 2\pi f L \quad \text{and} \quad X_C = \frac{1}{2\pi f_r C}$$

At the resonant frequency f_r of an LC circuit, the values of X_L and X_C are equal, so their formulae must also be equal.

$$X_L = 2\pi f L \quad = \quad X_C = \frac{1}{2\pi f_r C}$$

Multiplying both sides of the equation by $2\pi f_r C$ removes the fraction on the right and leaves just a single term of f (in the term 4 π 2fr2 LC) on the left.

$$4\pi^2 f_r^2 LC = 1$$

Dividing both sides of the result by $4\pi^2 LC$ leaves just f_r^2 on the left.

$$f_r^2 = \frac{1}{(4\pi^2 LC)}$$

Finally, taking the square root of both sides gives an equation for f_r and a useful formula for finding the resonant frequency of an LC circuit.

$$f_r = \frac{1}{(2\pi\sqrt{LC})}$$

However, although this formula is widely used at radio frequencies it is often not accurate enough at low frequencies where large inductors, having considerable internal resistance are used. In such a case a more complex formula is needed that also considers resistance. The formula below can be used for low frequency (large internal resistance) calculations.

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

The need for careful adjustment after circuit assembly is often a deciding factor for the discontinued use of pure LC circuits in many applications. They have been replaced in many applications by solid-state ceramic filters and resonating crystal tuned circuits that need no adjustment. Sometimes however, there may be a problem of multiple resonant frequencies at harmonics (multiples) of the required frequency with solid state filters. A single adjustable LC tuned circuit (that will have only one resonant frequency) may then also be included to overcome the problem.

Series Circuit Calculations.

In a series LCR circuit, especially at resonance, there is a lot happening, and consequently calculations are often multi stage. Formulae for many common calculations have been described in earlier modules in this series. The difference now is that the task of finding out relevant information about circuit conditions relies on selecting appropriate formulae and using them in a suitable sequence.

Continued.

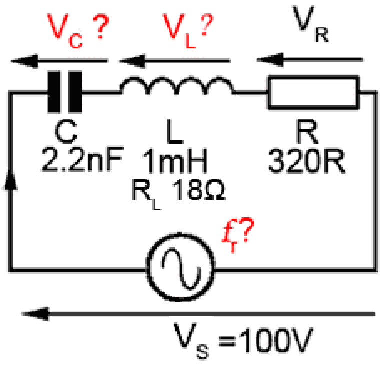
For example, in the problem below, values shown in red on the circuit diagram are required, but notice that V_C and V_L can't be worked out first, as a value for f_r (and another formula) is needed to calculate the reactance. Sometimes the task is made easier by remembering the 6 useful facts (page 6) about series resonance. In example 9.2.2 below there is no need to calculate both V_C and V_L because, at resonance X_C and X_L are equal, so calculate one and you know the other!

Notice however that V_L is not the same as the total voltage measured across L. The voltage across the internal resistance (at 90° to V_L) needs to be included, and because of the phase difference between V_L and the internal resistance voltage ($V\,R_L$), the total measurable inductor voltage $V_{L\,TOT}$ will be the phasor sum of V_L and $V\,R_L$.

Example 9.2.2 Series LCR Circuit Calculations.

For a series LCR circuit comprising:
L (1mH, internal resistance 18Ω,) C (2.2nF) and R (320Ω) connected to a 100V AC supply:
Calculate the resonant frequency and the maximum current.
Calculate the voltage V_C across C and the measurable voltage $V_{L\,TOT}$ across L.

1. Draw the circuit and list known values.
   L = 1mH
   R_L (internal R = 18Ω)
   C = 2.2nF
   R = 320Ω
   V_S = 100V



2. Decide on suitable formulae for unkown values.

(i) $f_r = \dfrac{1}{2\pi\sqrt{(LC)}}$

(ii) I_S (at f_r) $= \dfrac{V}{Z}$ or $\dfrac{V}{R_{TOT}}$ where $R_{TOT} = R + R_L$

(iii) $V_C = IX_C$ (and $X_C = \dfrac{1}{2\pi f_r C}$)

(iv) V_L $V_L = V_C$ at resonance f_r

(v) Total voltage across L ($V_{L\,TOT}$) $V_{L\,TOT} = \sqrt{(V_L^2 + V_{R_L}^2)}$

Work out each of these formulae (with pencil and paper and a calculator) remembering to work out the bracketed parts of the formula first, then check your answers by reading the text in in Module 9.3

Working this way while learning, is a good way to help understand how the maths work. There are of course a good many LCR calculators on the web but take a tip, WORK IT OUT FIRST, then try a web calculator (or more than one, as some are cleverer than others) to check your answer.

Module 9.3 Voltage Magnification

In the **answers** to the calculations in example 9.2.2 it should be noticeable that, at the circuit´s **resonant frequency f_r of 107kHz,** the reactive voltages across L and C are equal and each is greater than the circuit supply voltage V_S of 100V.

This is possible because, at resonance the **voltage (V_C = 199.56V)** across the capacitor, is in anti−phase to the **voltage (V_L = 199.56V)** across the inductance. As these two voltages are equal and opposite in phase, they completely cancel each other out, leaving only the supply voltage developed across the circuit impedance, which at resonance is the same as the total resistance of 320 + 18 = 338Ω.

At the resonant frequency the **current through the circuit is at a maximum value of about 296mA.** Because of the anti phase cancelling effect at resonance, the two reactive voltages V_C and V_L have "disappeared"! This leaves the supply current I_S effectively flowing through R and the inductor resistance R_L in series.

In this example the effect of the inductor´s 18Ω internal resistance on V_L is so small (0.03V) as to be negligible and **$V_{L\ TOT}$ is the same value as V_L at approximately 199.6V.**.

As the total circuit impedance is less than either the capacitive or inductive reactances at resonance, the supply voltage of 100V (developed across the circuit resistance) is less than either of the opposing reactive voltages V_C or V_L. This effect, where the internal component reactive voltages are greater than the supply voltage is called VOLTAGE MAGNIFICATION.

This can be a very useful property, and is used for example in the antenna stages of radio receivers where a series circuit, resonant at the frequency of the transmission being received, is used to magnify the voltage amplitude of the received signal voltage, before it is fed to any transistor amplifiers in the circuit.

The voltage magnification that takes place at resonance is given the symbol Q and the "Q Factor" (the voltage magnification) of LC Band Pass and Band Stop filter circuits for example, controls the "rejection", the ratio of the wanted to the unwanted frequencies that can be achieved by the circuit.

The effects of voltage magnification are particularly useful as they can provide magnification of AC signal voltages using only passive components, i.e. without the need for any external power supply.

In some cases voltage magnification can also be a dangerous property. in high voltage mains (line) operated equipment containing inductance and capacitance, care must be taken during design to ensure that the circuit does not resonate at frequencies too close to that of the mains (line) supply. If that should happen, extremely high reactive voltages could be generated within the equipment, with disastrous consequences for the circuit and / or the user.

The Q factor can be calculated using a simple formula. The ratio of the supply voltage V_S to either of the (equal) reactive voltages V_C or V_L will be in the same ratio as the total circuit resistance (R) is to either of the reactances (X_C or X_L) at resonance. The ratio of the reactive voltage V_L to the supply voltage V_S is the magnification factor Q.

The formula for finding Q (the voltage magnification) uses the ratio of the inductive reactance to the total circuit resistance.

Where X_L is the inductive reactance **at resonance**, given by $2\pi f_r L$ and R is the TOTAL circuit resistance. Note that Q does not have any units (volts, ohms etc.), as it is a RATIO

$$Q = \frac{X_L}{R} \text{ or } \frac{2\pi f_r L}{R}$$

Question: What is the magnification factor Q of the circuit in Example 9.2.2 in Module 9.2?

(No answer given, this one is down to YOU!)

Module 9.4 LCR Series Quiz

What you should know.

After studying Module 9, you should:

Be able to recognise LCR Series circuits and describe their action using phasor diagrams and appropriate equations.

Be able to describe LCR Series Circuits at resonance and the conditions for series resonance.

Be able to carry out calculations on LCR series circuits, involving reactance, impedance, component and circuit voltages and current.

Be able to Describe voltage magnification and calculate Q factor in LCR Series Circuits

Try our quiz, based on the information you can find in Module 9. Submit your answers and see how many you get right, but don't be disappointed if you get answers wrong. Just follow the hints to find the right answer and learn more about LCR Series Circuits and Resonance as you go.

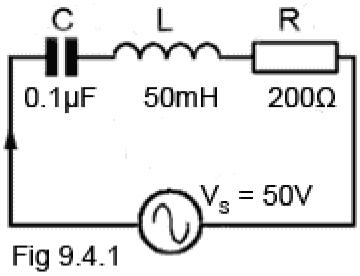


Fig 9.4.1

1.

With reference to Fig 9.4.1 the resonant frequency of the circuit will be approximately:

a) 71.2kHz

b) 444.3MHz

c) 2.251 kHz

d) 7.12MHz

2.

With reference to Fig 9.4.1, what will be the maximum supply current?

a) 70mA b) 250mA c) 500mA d) 14.14mA

3.

With reference to Fig 9.4.1, what will be the approximate voltage across C at resonance?

a) 177V b) 70V c) 1.7kV d) 353V

4.

With reference to Fig 9.4.1, what is the Q factor of the circuit?

a) 3.535V b) 1.4 c) 0.707 d) 3.5

Continued

5.

Which of the following statements about a series LCR circuit is true?

a) At resonance, the total reactance and total resistance are equal.

b) The impedance at resonance is purely inductive.

c) The current flowing in the circuit at resonance is at maximum.

d) The impedance at resonance is at maximum.

6.

If the values of L and C in a series LCR circuit are doubled, what will be the effect on the resonant frequency?

a) It will be halved.

b) It will not be changed.

c) It will double.

d) It will increase by four times.

7.

With reference to Fig 9.4.2, which phasor diagram shows a series LCR circuit at resonance?

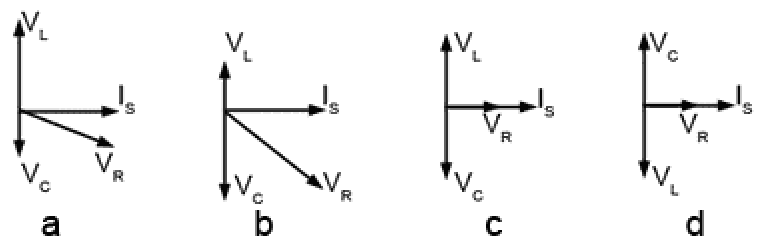


Fig 9.4.2

8.

What words are missing from the following statement? The impedance of series LCR circuit at resonance will be _______ and equal to the circuit _______ .

a) Minimum and resistance.

b) Maximum and resistance.

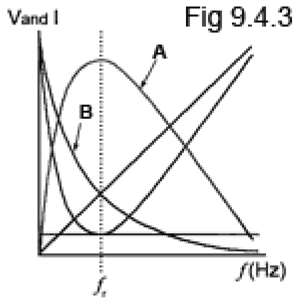
c) Minimum and reactance.

d) Maximum and reactance.

Continued

9.

With reference to the graph of voltages and current in a series resonant circuit shown in Fig 9.4.3, What quantity is represented by line A?



a) Circuit impedance.

b) Voltage across the capacitor.

c) Supply voltage.

d) Circuit current.

10.

With reference to the graph of voltages and current in a series resonant circuit shown in Fig 9.4.3, What quantity is represented by line B?

a) Circuit impedance.

b) Voltage across the capacitor.

c) Supply voltage.

d) Circuit current.

